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# On the Special Role of Deposits for Long-Term Lending\*

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## Abstract

In contrast to narrow banking proposals, I argue that deposits are a special form of financing, that makes banks more suitable to extend long-term loans when confronted with the risks of monetary policy. The synergy between deposit-taking and long-term lending arises because profits on deposits are highest after a contractionary monetary policy shock, precisely when the banks' balance sheets deteriorate due to maturity mismatch, and equity-constrained banks deleverage by cutting their lending. I quantify the impact of this mechanism in a dynamic bank model embedded in general equilibrium, and find that deposits mitigate the contraction of new lending at high interest rates by a factor between 25% and 50%.

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# 1 Introduction

Since the Great Financial Crisis of 2007-2009, amid discussions about how to reform the banking system, narrow banking proposals have received renewed interest. In a nutshell, what the different versions of narrow banking proposals have in common is the idea that the two main functions of current banks, deposit-taking and lending activities, should be separated in two different institutions. While the discussion has focused on the advantages of such proposals in terms of financial stability, in particular to avoid bank runs (see for example Cochrane (2004)), little has been said about the possible disadvantages.

What, if any, are the synergies between the deposit function and the lending function? This question is also important in the discussion about the transmission mechanism of monetary policy. While there is a literature about the "lending channel" of monetary policy (see Bernanke and Blinder (1988) and Kashyap and Stein (1994)), positing that, by affecting the supply of reserves and hence of deposits, monetary policy shifts the supply of loans, the more recent literature has internalized the Romer and Romer (2000) critique, which argues that if banks can switch without frictions to non-reservable forms of funding, deposit supply should not affect loan supply.

In this paper I argue instead that deposits are a special form of funding for banks that engage in maturity transformation: deposits do affect the supply of loans, as they provide banks with a natural hedge against the interest rate risk of monetary policy. I rely on the empirical results of Drechsel, Savov and Schnabl (2017), henceforth DSS. They show empirically that the spread between the Fed Fund rate and the deposit rate is increasing in the Fed Fund rate: for a percentage point increase in the Fed Fund rate, the interest rate on "core deposits" (defined as checking + saving + small time deposits) increases on average by only 40 bps, while at the same time deposit demand decreases by around 3%. While DSS focus on the fact that deposit demand decreases when the policy rate increases, which in their view amplifies the contraction in lending at high interest rates, I read the data from a different viewpoint and argue that *profits on deposits* are the important quantity. Profits on deposits, the product of the deposit spread and the quantity of deposits, strongly increase after a policy rate increase.

For a bank that borrows short-term and lends long-term, an interest rate increase

results in an erosion of equity. To the extent that banks face a friction in raising new equity on the market<sup>1</sup>, and that they are subject to an equity constraint (for economic or regulatory reasons), an erosion of bank equity leads to a contraction in bank lending. This is the basis of the “bank-capital channel” of monetary policy, see van den Heuvel (2003), henceforth vdH. The objective of my model is to show that profits on deposits, which increase after a policy rate increase, significantly mitigate the contraction in bank lending due to the bank-capital channel.

A confirmation of this mechanism comes from a recent paper by Carletti, De Marco, Ioannidou and Sette (2019). The authors exploit a tax reform in Italy that induced households and businesses to substitute bonds with bank deposits. Their main result is that banks that, as a result of the reform, experienced a larger increase in deposits, significantly increased the maturity of term loans, while not changing the overall credit supply. Thus, their findings support the idea that an increase in the quantity of deposits makes banks more willing to engage in maturity transformation.

In my model, banks borrow in the form of deposits and bonds. Each bank is a monopolist in the deposit market of its county, and pays an interest on deposits that is below the policy rate. Households can allocate the savings among three assets: cash, deposits and an asset – bonds – paying the policy rate. Cash (paying 0 interest) and deposits are “money-like” assets, as they reduce a transaction cost of consumption, as in Schmitt-Grohé and Uribe (2004), and, as in DSS, are imperfect substitutes. I show that this setup results in deposit spreads increasing in the policy rate. Intuitively, since cash, which is the main competitor of deposits as a money-like asset, always pays 0 interest, interest on deposits does not need to increase one-for-one with the policy rate.

Deposit demand decreases in the policy rate for two reasons. First, as the deposit spread decreases at higher policy rates, households choose to allocate more of their savings to the asset paying the policy rate. Second, as the policy rate increases, as a general equilibrium effect household consumption decreases, which in turn decreases the demand for money-like assets. Despite the lower deposit demand, however, profits on deposits are strongly increasing in the policy rate. Calibrating the model so that

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<sup>1</sup>Banks’ shareholders might also be unwilling to raise equity on the market, as this could disproportionately benefit bondholders. See Admati, Demarzo, Hellwig and Pfleiderer (2018).

both deposit spreads and deposit demand respond to a change in the policy rate in line with observed data, I obtain that profits on deposits almost triple when the policy rate goes from 2% to 6%.

On the asset side of the balance sheet, banks hold long-maturity loans to firms in a "Bank-dependent" (BD) sector. Banks are in monopolistic competition in the loan market. As in vdH, banks are risk-neutral and their objective is to maximize the present discounted value of future dividends, subject to an equity requirement constraint consistent with the Basel III Accord requirement. The financial friction in the model is that banks cannot raise equity on the market. They can retain profits, but this is expensive as accounting profits, i.e. operating income minus interest payments and write-offs on loans, are taxed at a constant tax rate  $\tau$ .

A contractionary monetary policy shock, affecting the long-duration asset side of banks more than the short-duration liability side, can erode the bank's equity and result in a violation of the equity constraint or in an increase of the probability of a future violation, thus affecting the bank's willingness to extend new loans. My objective is to quantify by how much the profits on deposits, which increase after a contractionary monetary shock, can mitigate the "bank capital channel", i.e. the contraction in bank lending at high policy rates due to balance sheet effects.

I make two alternative, extreme assumptions regarding the fixed cost of managing deposits. The first assumption ("zero-cost") is that this cost is zero. The second assumption ("high-cost") is that this cost is such that the average profit on deposits is equal to zero over time. My results indicate that deposits significantly mitigate the balance sheet effects on credit supply at high policy rates, and that this holds even in the "high-cost" case, thanks to the fact that profits on deposits are high when banks need them most.

The impact of the financial friction is very significant at high interest rates: when the interest rate is 1% above the natural rate, if a bank does not issue deposits new lending is on average 9.5% lower than in the model without the financial friction. With deposits, the contraction in new lending is 7% in the "high-cost" case, and only 5% in the "zero-cost" case.

When the interest rate is 2% above the natural rate, without deposits new lending is on average 24.5% lower than in the model without financial friction. With deposits,

in the “high cost” case the contraction in new lending is 18%, and in the “zero-cost” case it is only 13%.

In sum, deposits mitigate the contraction in new lending by a factor roughly between 25% and 50% when the policy rate is at least 1% above the natural rate, depending on the assumption about their managing cost.

An evaluation of the impact of bank credit supply on output is beyond the scope of this paper. However, based on the estimates by Capiello, Kadareja, Kok and Protopapa (2010), credit supply has a significant impact on output in the euro area<sup>2</sup>. Combining my results on credit supply with their estimates, I find that a narrow banking reform, by preventing banks that extend loans from issuing deposits, might reduce output in the euro area by up to 45 bps when the policy rate is 1% above the natural rate and by up to 1% when the policy rate is 2% above the natural rate.

To sum up, the contribution of this paper is threefold: first, I identify a new mechanism by which deposits affect banks’ loan extension and the transmission of monetary policy. Second, I quantify the impact of deposits by embedding this mechanism in a bank model inspired by the partial equilibrium model of vdH. Third, I embed the bank model in general equilibrium, which allows me to take into account the effect of changes in the macroeconomic environment, in particular changes in aggregate consumption and inflation, on the bank’s lending problem.

## Literature Review

This paper contributes to the literature on the role of bank lending in the transmission of monetary policy, often called “the credit channel” of monetary policy.

A vast part of this literature (see, for example, Gertler and Gilchrist (1993), Bernanke and Gertler (1995), Bernanke, Gertler and Gilchrist (1999)) focuses on the so-called “broad credit channel”, or “financial accelerator”: an interest rate increase causes a deterioration of borrower firms’ balance sheet, which in turn causes an increase in the external finance premium and a decrease in loan demand over and above the decrease due purely to the risk-free interest rate increase.

Another part of the literature (see, for example, Bernanke and Blinder (1988),

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<sup>2</sup>This result is obtained using the same methodology as Driscoll (2004), who instead finds insignificant impact of credit supply on output in the US.

Kashyap and Stein (1994)) focuses on the "narrow credit channel", or "lending channel": a decrease in central bank reserves forces banks to issue fewer reservable deposits. Assuming that deposits are the main source of funding for bank loans, lower deposit issuance would result in lower loan issuance. As previously discussed, this line of reasoning has been criticized by Romer and Romer (2000).

Other branches of the literature find links between monetary policy and bank loan supply through different channels: for example the previously mentioned "bank capital channel" (van den Heuvel (2003), Adrian and Shin (2010a)) and the "risk-taking channel" (Borio and Zhu (2012) and Adrian and Shin (2010b)), according to which monetary policy may influence banks' perception of risk or attitude toward risk.

This paper is closer to the "lending channel" literature in that it focuses on the role of deposits and on the effect of monetary policy on loan supply, and to the "bank capital channel" literature, in that it argues that deposits have an impact on this specific channel.

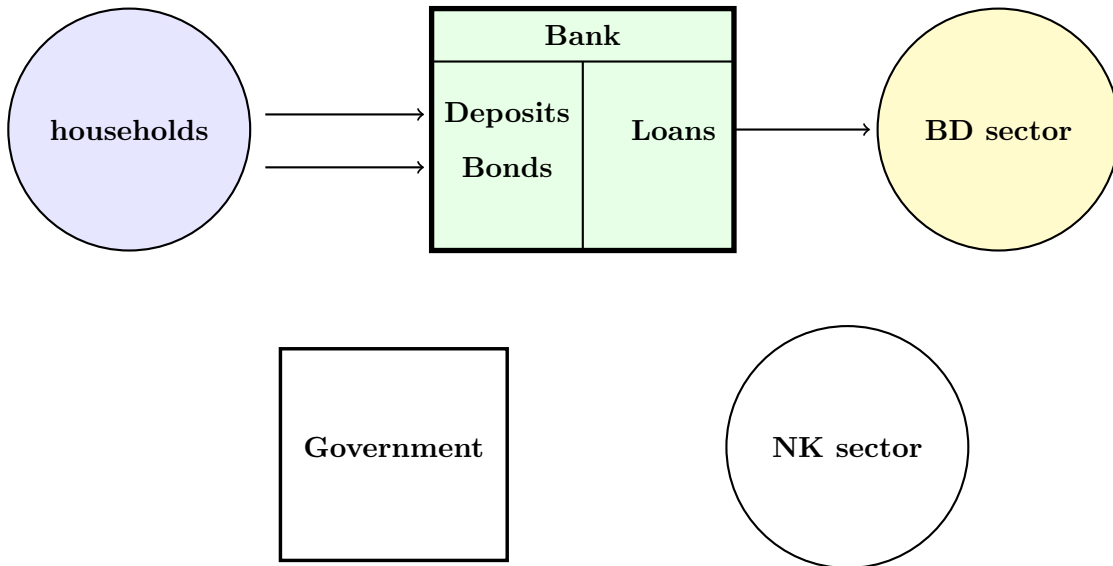
In contrast to the lending channel literature, however, this paper argues that deposits have a mitigating, rather than amplifying, effect on loan supply, the main difference being that instead of focusing on the quantity of deposits, that decreases after an interest rate increase, I focus on deposit profits, that increase after an interest rate increase.

Some recent papers emphasizing that a change in the policy rate may have an effect on banks' interest rate margins and hence affect banks' profits on new business are Brunnermeier and Koby (2018), Di Tella and Kurlat (2017) and Wang, Whited, Wu and Xiao (2018). The setup in Wang, Whited, Wu and Xiao (2018) is the most similar to this paper. One of their results, partly overlapping with this paper, is that banks' market power interacts with the friction due to capital requirements. In particular they find that there is a "reversal interest rate", below which reduced bank profits make a further rate cut contractionary. Eggertsson, Juelsrud, Summers, and Wold (2019) also find a similar result: they argue that, when the policy rate is in negative territory, negative profits on deposits negatively affect bank lending. My paper finds instead that the effect of deposits is more important in higher rate territory, when deposits profits are higher. Moreover, differently from these papers, my specific mechanism relies on the ability of deposits to reduce the risk of maturity transformation.

## 2 Model Overview

The agents in the model and the credit flow in the economy are shown in Figure 1. The central agents in the model are in the top row of the figure. In addition to banks, the other important agents are households, that generate deposit demand, and firms in the “Bank-Dependent (BD) sector” (which need to borrow from banks), that generate loan demand. The agents in the bottom row, the government and firms in the “New Keynesian (NK) sector”, allow the embedding of the model in General Equilibrium: the government sets the policy rate, the central stochastic quantity in the economy, following a Taylor rule; firms in the NK sector generate the New Keynesian Phillips Curve (NKPC).

**Figure 1**



Section 3 of the paper focuses on households and banks in their deposit-taking function. These are the agents that determine deposit demand and supply. As described in the Introduction, households allocate their savings among three assets: cash, deposits and an asset paying the policy rate. Cash and deposits, despite paying low interest, are held by the household for their money-like properties.

Section 4 of the paper focuses on the bank’s problem in its loan-extending function, with the objective of investigating how the bank’s lending decisions are affected by the



profits on deposits. In the Appendix I describe in detail the two firm sectors, that correspond to two alternative technologies for the production of the unique consumption/investment good. In particular, the “NK sector”, similar to firms in the basic New-Keynesian model in Gali (2008), is useful to generate the NKPC.

One comment is in order on the interaction between the NK sector, the banks and the BD sector in my model. Embedding the bank problem, which is highly non-linear, in the general equilibrium NK model is a non-trivial challenge. To make the problem simpler, I assume that both the household disutility of labor and the production function in the NK sector are linear in labor. I show that this implies that the Euler equation, the New-Keynesian Phillips curve and the Taylor rule form a system of three equations in three variables, consumption, inflation and the policy interest rate, independent of banks (except for the choice of the deposit rate) and of production in the BD sector. The intuition for why this happens is that, with linear labor disutility and linear production function in the New-Keynesian sector, households would be able to adjust their labor supply and thus production in the New-Keynesian sector to compensate for fluctuations in the production of the bank-dependent sector. Although with these assumption the model would be unsuitable to evaluate the impact of the banking sector on the macroeconomy, it is calibrated to generate a realistic response of consumption and inflation to monetary shocks, very similar to that of the basic NK model in Gali (2008), and is thus suitable to analyze the effect of the macroeconomic environment on the bank’s problem, which is the focus of this paper.

## 3 The Economics of Deposits

### 3.1 Empirical Evidence

DSS present detailed empirical evidence on the relationship between deposit quantities and the Fed fund rate, and between deposit spreads and the Fed Fund rate. They estimate that an increase of 100 bps in the Fed Fund rate leads on average to a 61 bps increase in the deposit spread. The increase in the spread is shown to be clearly correlated to banks’ market power: within the same bank, branches in high-concentration areas increase their spread by less than branches in low-concentration areas.

They also estimate the semi-elasticity of deposits with respect to the deposit spread to be -5.3. Thus, an increase of 100 bps in the Fed Fund rate, which is estimated to raise the deposit spread by 61 bps, would induce a decrease in the quantity of deposits of around 320 bps.

A clear implication, which is not drawn by DSS, is that profits on deposits increase after a Fed fund increase. For example, an increase of the Fed Fund rate from 4% to 5% represents a 25% increase in the Fed Fund rate rate, which also translates in a 25% increase in the deposit spread if the relationship between the latter and the Fed Fund rate is approximately linear. Despite the 3% decrease in the quantity of deposits, profits on deposits (equal to the product of deposit spread and deposit quantity) would increase by over 20%. Only for extremely high values of the Fed Fund rate (over 30%!) the decrease in the deposit quantity would be more important than the increase in the spread.

### 3.2 Model: Households and Deposit Demand

Households consume, work and save in three different nominal assets: an asset  $A_t$  (government or bank bonds), bank deposits  $D_t$  or cash  $M_t$ . Each of the three saving instruments pays a different interest rate: cash pays zero interest, deposits pay an interest  $i_t^d$  and the asset  $A$  pays the policy rate  $i_t$ . The policy rate follows a stochastic process and is set by the government, as we will see later. Households maximize

$$U = \sum_{t=0}^{\infty} \beta^t E_0[u(c_t, h_t)] \quad (1)$$

where  $c_t$  is consumption and  $h_t$  hours worked. The intra-period utility function is separable in consumption and labor

$$u(c, h) = \frac{c^{1-\sigma}}{1-\sigma} - F(h) \quad (2)$$

<sup>3</sup> subject to the budget constraint

$$\begin{aligned} & W_t h_t + (1 + i_{t-1})A_{t-1} + (1 + i_{t-1}^d)D_{t-1} + M_{t-1} + P_t \Pi_t \\ &= P_t c_t (1 + \chi_t(x_t)) + D_t + A_t + M_t \end{aligned} \quad (3)$$

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<sup>3</sup>In order to embed the model in General Equilibrium in a simple way, I use a linear disutility of labor  $F(h) \propto h$ . This will be explained in detail in Appendix B1. It has no impact on the demand for deposits, that is the focus of this section.

where  $W_t$  is the (nominal) wage,  $\Pi_t$  are real profits from firms and banks and  $P_t$  is the price level of the consumption good. Households face a proportional transaction cost of consumption  $\chi_t$ , which depends on the ratio  $x_t$  between nominal consumption and liquidity  $l_t$ , a bundle of cash  $M_t$  and bank deposits  $D_t$ :

$$x_t \equiv \frac{P_t c_t}{l_t} \quad (4)$$

$$l_t \equiv \left( \delta M_t^{\frac{\epsilon-1}{\epsilon}} + D_t^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \quad (5)$$

$x_t$  can be interpreted as the velocity of liquid assets. As in DSS,  $\delta$  measures the liquidity of cash relative to deposits and  $\epsilon$  is the elasticity of substitution between cash and deposits. Appendix A1 contains all the first-order conditions of the household problem.

### 3.2.1 The choice among the three assets

From the Euler equations with respect to cash and deposits, I obtain that the ratio between cash and deposit holdings is a function of the deposit spread relative to the policy rate,  $\frac{i_t - i_t^d}{i_t}$ , and is independent of the transaction cost  $\chi(x)$

$$\frac{M_t}{D_t} = \left( \delta \frac{i_t - i_t^d}{i_t} \right)^{\epsilon} \quad (6)$$

This also implies that liquidity can be written as

$$l_t = f D_t \quad (7)$$

with

$$f \equiv \left( 1 + \delta \left( \frac{i_t - i_t^d}{i_t} \right)^{\epsilon-1} \right)^{\frac{\epsilon}{\epsilon-1}} \quad (8)$$

From the Euler equation with respect to deposits and with respect to the risk-free asset I obtain the equation

$$f^{\frac{1}{\epsilon}} x_t^2 \chi'(x_t) = \frac{i_t - i_t^d}{1 + i_t} \quad (9)$$

which allows us to find the demand for liquidity  $l_t = \frac{P_t c_t}{x_t}$  and the demand for deposits  $D_t = f^{-1} l_t$  as a function of  $i_t$  and  $i_t^d$ , for a given level of nominal consumption  $P_t c_t$ . The meaning of (9) is simple: for a given level of consumption, liquidity holdings are

chosen so that the marginal benefits in terms of reduction of the transaction cost (LHS) equate the marginal cost, in terms of forgone interest (RHS).

I now specialize the transaction cost to the form used by Schmitt-Grohé and Uribe (2004)

$$\chi(x) = ax + \frac{b}{x} - 2\sqrt{ab} \quad (10)$$

This function has a minimum for  $x = \sqrt{\frac{b}{a}}$ , which represents the satiation level of liquidity.<sup>4</sup> Inserting (10) in (9) I obtain

$$x = \sqrt{\frac{f^{-\frac{1}{\epsilon}}(i_t - i_t^d) + b(1 + i_t)}{a(1 + i_t)}} \quad (11)$$

from which it is easy to obtain the deposit demand

$$D_t = \frac{P_t c_t}{f} \sqrt{\frac{a(1 + i_t)}{f^{-\frac{1}{\epsilon}}(i_t - i_t^d) + b(1 + i_t)}} \quad (12)$$

### 3.3 Deposit Rate Determination

At time  $t$  a monopolist bank issues deposits  $D_t$  and sets the deposit rate  $i_t^d$ . The deposit demand as a function of the deposit rate is (12). The objective of the bank is to maximize the profits it will realize next period, which are, in real terms

$$\Pi_{t+1} = (i_t - i_t^d) \frac{D_t}{P_{t+1}} \quad (13)$$

taking the policy rate  $i_t$ , consumption  $c_t$  and inflation  $\frac{P_{t+1}}{P_t}$  as given. The demand for deposit  $D_t$  is given by (12). Appendix A2 shows the first-order condition for  $i_t^d$ . There I also show that for  $i_t \ll b$  the optimal spread is

$$i_t - i_t^d = \left( \frac{1}{(\epsilon - 1)\delta^\epsilon} \right)^{\frac{1}{\epsilon-1}} i_t \quad (14)$$

whereas for  $i_t \gg b$

$$i_t - i_t^d = \left( \frac{1}{2(\epsilon - 1)\delta^\epsilon} \right)^{\frac{1}{\epsilon-1}} i_t \quad (15)$$

showing that the deposit spread is indeed increasing in the policy rate.

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<sup>4</sup>Taken literally, this transaction cost increases for liquidity holdings bigger than the satiation level. This has no consequence because liquidity holdings bigger than satiation level will never be chosen.

### 3.4 Consumption and Inflation Determination

A full description of the economics of deposits, including the quantity of deposits and the banks' profits on deposit as a function of the policy rate, requires knowledge of aggregate consumption, affecting the demand for deposits and hence the banks' profits on deposits, and inflation, also affecting the profits on deposits.

The full model including government and firms results in a system of three equations: the Euler equation, the Taylor rule, which describes how the government sets the policy rate, and the New-Keynesian Phillips Curve (NKPC), obtained from firms in the "NK sector", plus an equation describing the stochastic process followed by a "monetary shock". The solution of this system gives consumption  $c_t$ , the policy rate  $i_t$  and inflation  $\pi_t$  as a function of the shock  $v_t$ , or, equivalently, consumption and inflation as a function of the policy rate  $i_t$ . Details about firms in the NK sector and about the derivation of the NKPC are given in Appendix B1.

Using the velocity equation (11) and the deposit rate  $i^d = i^d(i_t)$ , obtained as a function of the policy rate as the solution of (54), the Euler equation (51) can be written in log-linear form around the steady state

$$\hat{c}_t = E_t[\hat{c}_{t+1}] - \frac{1}{\sigma}((1+p)(i_t - r_n) - \mathbb{E}_t[\pi_{t+1}]) + \frac{p}{\sigma}E_t[i_{t+1} - r_n] \quad (16)$$

$r_n \equiv \beta^{-1} - 1$  is the steady state value of the policy rate,  $\hat{c} \equiv \frac{c_t - c^{SS}}{c^{SS}}$  ( $c^{SS}$  is steady-state consumption) and

$$p \equiv \frac{2a\bar{x}'}{(1 + 2a\bar{x} - \sqrt{ab})} \quad (17)$$

where  $\bar{x}$  is the value of velocity (11) at  $i_t = r_n$  and  $\bar{x}'$  is the derivative of velocity with respect to  $i_t$  at  $i_t = r_n$ .

The policy rate  $i_t$  is set according to the Taylor rule

$$i_t = r_n + \phi_\pi \pi_t + \phi_c c_t + v_t \quad (18)$$

The monetary shock  $v_t$  follows an auto-regressive process

$$v_{t+1} = \rho_v v_t + \epsilon_{t+1}^v \quad (19)$$

Finally, the NKPC is

$$\pi_t = \beta E_t[\pi_{t+1}] + \Lambda \hat{c}_t \quad (20)$$

The slope of the Phillips curve  $\Lambda$ , as seen in Appendix B1, is related to parameters of the NK firms, in particular to the degree of price stickiness faced by these firms.

The system (16), (18), (19) and (20) implies that the inflation rate and consumption (in log-deviation from steady state) are proportional to the policy rate  $i_t$  (in deviation from the steady state value  $r_n$ ):

$$\begin{aligned}\hat{c}_t &= b_c(i_t - r_n) \\ \pi_t &= b_\pi(i_t - r_n)\end{aligned}$$

where

$$\begin{aligned}b_c &= -\frac{1 + p(1 - \rho_v)}{\sigma(1 - \rho_v) - \rho_v \frac{\Lambda}{1 - \beta\rho_v}} \\ b_\pi &= -\frac{1 + p(1 - \rho_v)}{\sigma(1 - \rho_v) \frac{1 - \beta\rho_v}{\Lambda} - \rho_v}\end{aligned}$$

### 3.5 Economics of Deposits: Calibration and Results

The parameters that are relevant for the economics of deposits are shown in Table 1. On the left-hand side of the table we find the parameters related to household preferences and money demand. The parameters  $a$  and  $b$  appearing in the transaction cost have been calibrated to the US economy by Schmitt-Grohé and Uribe (2004), using quarterly data from 1960 to 2000<sup>5</sup>. The elasticity of substitution between cash and deposits,  $\epsilon_L$ , and the liquidity of cash relative to deposits  $\delta$  are such that for a one percent increase in the policy rate, the increase in the deposit rate ranges from 35 bps (when the policy rate is close to zero) to 55 bps (when the policy rate is high). On the right-hand side of the table we find the parameters of the Taylor rule and the New-Keynesian Phillips curve. These are close to the parameters in Gali (1983). The slope of the NKPC, as we see in (67) and (68), is essentially related to the parameter  $\alpha$  of the production function of NK firms, that I take equal to 0 for reasons explained in Appendix B1, and the price stickiness Calvo parameter  $\theta$ , that I take equal to 0.75, implying that prices are reset once per year. With these parameters, 1% increase in the

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<sup>5</sup>To transform the parameters  $a$  and  $b$  estimated by Schmitt-Grohé and Uribe (2004), which are appropriate when annual consumption is used, into the corresponding parameters that are appropriate when using quarterly consumption, I multiply their value of  $a$  by 4 and divide their value of  $b$  by 4.

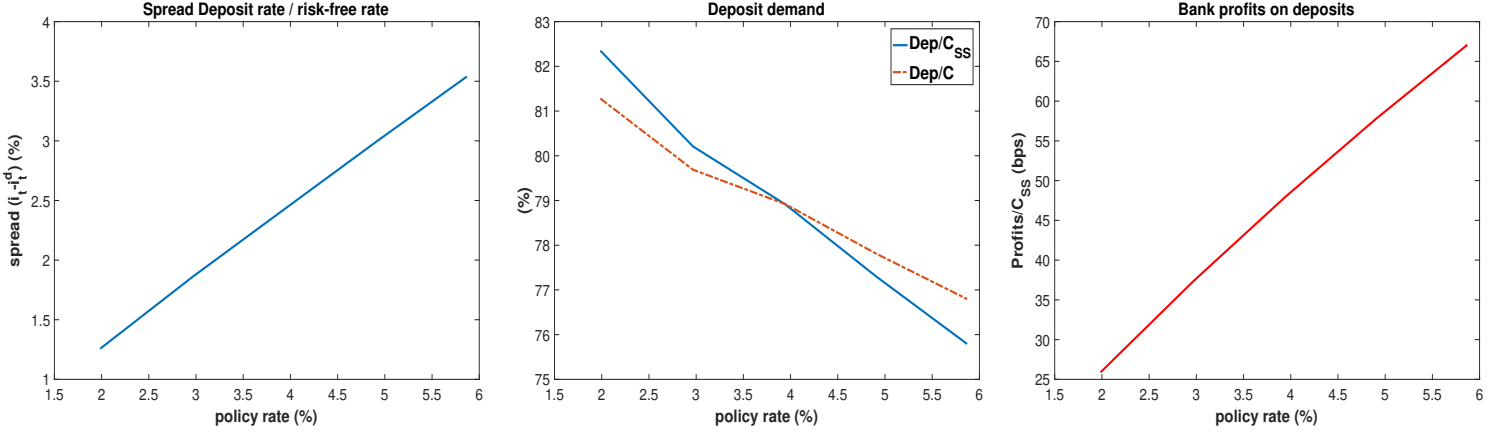
policy rate results in a decrease in consumption of 0.68% and a decrease in inflation of 0.87%, close to the behavior of these variables in the basic NK model of Gali (1983).

Table 1 Calibration: Economics of Deposits

Household & Money Demand Parameters		Taylor rule and NKPC parameters	
Parameter	Description	Parameter	Description
$\beta = 0.99$	discount rate	$\rho_v=0.6$	persistence of monetary policy shock
$\sigma = 1.5$	elasticity of intertemporal substitution	$\phi_c=0.5/4$	Taylor rule parameter
$\delta = 1.05$	liquidity of cash relative to deposits	$\phi_\pi = 1.5$	Taylor rule parameter
$\epsilon_L = 3$	elast. of subst. cash/deposits	$\Lambda = 0.086$	slope of the NKPC
$a = 0.0111 \times 4$	transaction cost parameter		
$b = 0.07524/4$	transaction cost parameter		

The three panels of Figure 1 show the main results related to deposits. The panel on the left shows that the deposit spread,  $i - i^d$ , grows from 127 bps when  $i = 2\%$  to 365 bps when  $i = 6\%$ . The spread grows approximately linearly in this range, by about 60 bps for every 1% increase in the policy rate, in line with the empirical findings by DSS. The center panel shows that the quantity of deposits is decreasing in the policy rate. For a 1% increase in the policy rate, the quantity of deposits decreases by 2.86% in this range. It decreases through two channels: first, since the deposit spread increases, households prefer to allocate more of their savings to the asset  $A$ , and they lower their holdings of deposits as a fraction of consumption (see the dashed line in the center panel). Second, household consumption decreases after a contractionary shock, so households need to hold less deposits. The solid line in the center panel shows that the quantity of deposits as a fraction of steady state consumption decreases even more. All in all, however, when  $i = 6\%$  the quantity of deposits is only 11% smaller than when  $i = 2\%$ , while the spread increases by almost a factor 3. The right panel shows that profits on deposits at  $i = 6\%$  are higher than at  $i = 2\%$  by a factor 2.6.

Figure 1: Deposits



## 4 The Banks' Problem

In this section I outline the decision problem of the bank. There is a continuum of banks of measure 1, each of which borrows in the form of deposits and bonds and extends loans. The key friction is that banks cannot raise equity on the market, they can only build equity by retaining profits. Moreover, banks need to satisfy a minimum equity requirement, as described below.

Some elements of the structure of the bank's problem, notably the financial friction and the equity requirement, are similar to vdH. The objective of vdH was to evaluate the impact of the friction by comparing the lending behavior of the firm facing the friction to the behavior of the "unconstrained bank". The latter is a bank that can costlessly raise equity on the market, or that holds enough equity that the constraint does not currently bind and will never bind in the future. In contrast, the main purpose of this paper is to evaluate the impact of deposits on the lending behavior of the constrained bank. I will compare the lending behavior of the constrained bank that optimally chooses deposits to that of the constrained bank that borrows fully in bonds, against the benchmark of the unconstrained bank. The latter is able to exploit all profitable opportunities and its lending is not affected by deposits.

I assume that each bank is a monopolist in the deposit market, and is in monopolistic competition with other banks in the loan market. We can imagine for example that each bank operates in a county, and regulation (or high transportation costs) prevent



households in one county from holding deposits in another county. Firms however are allowed to take loans from banks outside their county. A bank's balance sheet is

Assets	Liabilities
$P_t L_t$	$D_t$
	$B_t$
	$P_t E_t$

The following subsections describe in detail the components of the balance sheet.

## 4.1 Loans

Loans are long-term, risky real assets. A loan granted at time  $t$  is a security bought by bank  $i$  at price  $\mathcal{P}_t^i$ , that by contract demands a real payment  $(\bar{\delta} + \bar{\rho})(1 - \bar{\delta})^{n-1}$  from the borrower to the bank at each time  $t + n$ ,  $n = 1, \dots, \infty$ .<sup>6</sup>

In other words, every period the borrower is required to repay a fraction  $\bar{\delta}$  of the outstanding principal, and a constant real interest rate  $\bar{\rho}$ . The contractual loan duration is therefore  $\frac{1}{\bar{\delta}}$ .

However, loans carry default risk: a (stochastic) fraction  $\omega_t$  of the outstanding loans defaults at each time  $t$ . Hence, a loan granted at time  $t$  results in the following actual payments

$$\begin{aligned}
& (\bar{\delta} + \bar{\rho})(1 - \omega_{t+1}) && \text{at } t + 1 \\
& (\bar{\delta} + \bar{\rho})(1 - \omega_{t+2})(1 - \bar{\delta} - \omega_{t+1}) && \text{at } t + 2 \\
& \dots && \\
& (\bar{\delta} + \bar{\rho})(1 - \omega_{t+n})(1 - \bar{\delta} - \omega_{t+n-1}) \dots (1 - \bar{\delta} - \omega_{t+2})(1 - \bar{\delta} - \omega_{t+1}) && \text{at } t + n
\end{aligned} \tag{21}$$

I take the default shocks  $\omega$  to be i.i.d. and independent of the other shock in the economy, the monetary shock. One loan granted at time  $t$  is an identical security, at

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<sup>6</sup>Appendix B2 shows that an equivalent formulation is possible, in term of “unit loans”: one unit loan granted at time  $t$  involves one unit of good transferred from the bank to the borrowing firm at  $t$ , and demands a real payment  $(\bar{\delta} + \rho^{(t)})(1 - \bar{\delta})^{n-1}$  from the borrower at each subsequent time  $t + n$  (the loan rate  $\rho^{(t)}$  is decided at time  $t$  but is constant over the life of the loan).

$t+1$ , to  $(1 - \bar{\delta} - \omega_{t+1})$  loans granted at  $t+1$ . I call  $L_t$  the outstanding loans at time  $t$  (a state variable), and  $N_t$  the number of new loans granted at time  $t$  (a decision variable). Loans evolve according to

$$L_{t+1} = (1 - \bar{\delta} - \omega_{t+1})(L_t + N_t) \quad (22)$$

Loan demand comes from firms in the *BD sector*. These firms view loans from different banks as imperfect substitutes, which implies that banks are in monopolistic competition in the loan market. This imperfect substitutability might be due to bank specialization in the financing and monitoring of different activities, or to geographical specialization. Paravisini, Rappoport and Schnabl (2017) provide empirical evidence of such specialization.

The production function of firms in the BD sector, described in detail in Appendix B3, generates the loan demand curve

$$N_t^i = (\zeta \nu)^{\frac{1}{1-\nu}} (\bar{\rho} + \bar{\delta})^{-\frac{1}{1-\nu}} (\mathcal{P}_t^i)^{\epsilon_B - 1} (\mathcal{P}_t^M)^{\frac{1}{1-\nu} - \epsilon_B} \quad (23)$$

Here  $\mathcal{P}_t^i$  is the loan price obtained by bank  $i$  at time  $t$ ,  $\zeta$  and  $\nu$  are parameters of the BD firms production function,  $\epsilon_B > 0$  is the elasticity of substitution of loans from different banks, and  $\mathcal{P}_t^M$  is the “market loan price” at time  $t$  (details in Appendix B2). The most important element in (23) is the fact that the demand for loans of bank  $i$  is upward sloping in the loan price paid by the same bank:  $\mathcal{P}_t^i(N_t^i)$  is an increasing function. The more the banks lends, the lower is the expected return on loans. This results from imperfect competition.

## 4.2 Liabilities: Deposits and Bonds

Deposits  $D_t$  are one-period securities issued at time  $t$ , yielding an interest  $i_t^d$  chosen by the bank at time  $t$ . The downward sloping demand curve (taking households consumption as given) is given by (12).

Bonds  $B_t$  are one-period securities yielding the policy rate  $i_t$ , set by the government.<sup>7</sup>

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<sup>7</sup>I assume that the bank ends its activity when  $E_t < 0$ , and the bonds are guaranteed by the government. This guarantees that the bank can borrow at the risk-free rate.

### 4.3 Equity

At the beginning of period  $t$ , before the bank makes new loan, deposit, and dividend decisions, the bank's equity is equal to the value of the outstanding loans, plus the cashflows coming from assets and liabilities

$$E_t = V_t^L L_t + CF_t \quad (24)$$

All terms in (24) are in real terms. I take the value of a loan  $V_t^L$ , used by the regulator to compute the accounting value of the bank's equity, to be equal to the highest price that a bank would be willing to pay for an identical loan, i.e. the price that an unconstrained bank would pay. I call this price  $\mathcal{P}_t^U$ . It can be found by solving the problem of the unconstrained bank. It is a decreasing function of the interest rate and its value in my calibration can be found in Table 4. Although in this model there is no secondary market for loans, this choice is in the spirit of "mark-to-market accounting", which is the current accounting standard.<sup>8</sup>

Cashflows  $CF_t$  include cashflows from assets at time  $t$ , i.e. coupons and principal repayments, the repayment of liabilities, taxes  $T_t$  and fixed costs  $c_F < 0$

$$CF_t = (\bar{\delta} + \bar{\rho})(1 - \omega_t)(L_{t-1} + N_{t-1}) - (1 + i_{t-1})\frac{B_{t-1}}{P_t} - (1 + i_{t-1}^d)\frac{D_{t-1}}{P_t} + c_F - \frac{T_t}{P_t} \quad (25)$$

where  $B_{t-1}$  and  $D_{t-1}$  are the bonds and deposits issued at time  $t-1$ , due at  $t$ . Taxes  $T_t$  are a fraction  $\tau$  of the taxable base  $TB_t$ , given by the interest received on the loans, net of the interest paid on liabilities and the fixed costs:

$$T_t = \tau \times TB_t = \tau \times [P_t(\bar{\rho}(1 - \omega_t)(L_{t-1} + N_{t-1}) + c_F - \omega_t(L_{t-1} + N_{t-1})) - i_{t-1}B_{t-1} - i_{t-1}^d D_{t-1}] \quad (26)$$

The bank then makes a decision about the dividends  $Div_t$  to distribute in period  $t$ , the new loans  $N_t$  to purchase at price  $\mathcal{P}_t(N_t)$  and the liabilities  $B_t$  and  $D_t$ . In order to honor the previous-period liabilities and carry out the decision about  $Div_t$  and  $N_t$ ,  $B_t$  and  $D_t$  need to satisfy

$$\begin{aligned} D_t + B_t &= P_t(Div_t + \mathcal{P}_t(N_t)N_t - CF_t) \\ &= P_t(Div_t + \mathcal{P}_t(N_t)N_t - E_t + V_t^L L_t) \end{aligned} \quad (27)$$

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<sup>8</sup>The Financial Accounting Standard 157 (FAS157) introduced by the Financial Accounting Standard Board (FASB) in 2006, established that assets should be valued in agreement with their "exit price", replacing the previously used "historical-cost" accounting.

(where the last equality uses (24)), so that at the end of the period the bank's equity, equal to the value of the assets minus the liabilities, can be written as

$$E'_t = V_t^L(L_t + N_t) - B_t - D_t = E_t - Div_t + (V_t^L - \mathcal{P}_t(N_t))N_t \quad (28)$$

Interestingly, (28) shows that reducing  $N_t$  is a way to immediately rebuild equity: since  $\mathcal{P}_t(N_t)$  is upward sloping, by reducing the supply of new loans the bank can purchase the new loans at a price which is lower than their accounting value  $V_t^L$ .

## 4.4 Equity Requirement

I model the equity requirement following vdH. All loans are assumed to be in the highest risk category. At the beginning of a period  $t$ , a bank is free to issue new loans and pay dividends only if the value of its equity exceeds the regulatory minimum. Moreover, if new loans are issued and/or dividends are paid, the end-of period value of equity must still be equal to or above the regulatory minimum. Thus, the requirement can be expressed in two statements

- If, at the beginning of period  $t$ ,  $E_t < \gamma V_t^L L_t$ , then it must be  $N_t = Div_t = 0$ .
- Otherwise,  $N_t$  and  $Div_t$  must be such that at the end of the period  $E'_t \geq \gamma V_t^L (L_t + N_t)$ .

## 4.5 Bank's objective

The bank is a risk-neutral entity whose value function is given by the present discounted value of future dividends. It can be written in recursive form

$$V_t(E_t, L_t, i_t) = \max_{\{N_t, Div_t, D_t\}} Div_t + \frac{1}{1 + i_t} \mathbb{E}_t[V_{t+1}(E_{t+1}, L_{t+1}, i_{t+1})] \quad (29)$$

subject to

- The law of motion of loans (22);
- The law of motion of equity

$$E_{t+1} = V_{t+1}^L(i_{t+1})(1 - \delta - \omega_{t+1})(L_t + N_t) + CF_{t+1} \quad (30)$$

with

$$\begin{aligned}
CF_{t+1} &= (\bar{\delta} + (1 - \tau)\bar{\rho})(1 - \omega_{t+1})(L_t + N_t) - (1 + (1 - \tau)i_t)\frac{B_t + D_t}{P_{t+1}} \\
&+ (1 - \tau)(i_t - i_t^d)\frac{D_t}{P_{t+1}} + (1 - \tau)c_F + \tau\omega_{t+1}(L_t + N_t)
\end{aligned} \tag{31}$$

As shown in (27) the sum  $B_t + D_t$  can be written in terms of the decision variables  $Div_t$  and  $N_t$ . Hence time- $t + 1$ -equity depends on time- $t$  state variables, time- $t$  decision variables ( $Div_t$ ,  $N_t$  and  $D_t$ ) time- $t + 1$  shocks.

- The deposit demand function (12), which also depends on aggregate consumption  $c_t$ , an endogenous macroeconomic variable;
- The equity requirement constraint;
- The loan demand function (23).

Notice that the decision about deposits  $D_t$  (deposits) only affects the bank through its contribution to next period's cashflows (31) and hence equity (30). Since  $B_t + D_t$ , as already pointed out, can be expressed in terms of the decision variables  $N_t$  and  $Div_t$ , deposits affect the bank's problem only through the contribution to next period's cashflows

$$(1 - \tau)(i_t - i_t^d)\frac{D_t}{P_{t+1}} \tag{32}$$

This shows that despite the complexity of the bank's decision problem, the choice about deposits is a static one, and the deposit rate (and hence deposit quantity) chosen by the bank are, for each value of the policy rate, those obtained in section 2.

Notice also that, with perfect competition in the lending market, and given banks' risk neutrality, only unconstrained banks would lend. Constrained banks would have no incentive to lend, given that they would make no profits in expectation, and that they would risk violating the equity requirement, with the consequence of not being able to distribute dividends.

## 4.6 The Unconstrained Value Function

The value function and policy functions of the unconstrained bank are the benchmark against which we can compare the policy functions of the constrained bank. As in vdH,

the value function of the unconstrained bank has the form

$$V(E_t, L_t, i_t) = a_0(i_t) + E_t + a_L(i_t)L_t \quad (33)$$

which can be verified by inserting (33) in (29), and maximizing with respect to  $N_t$ . By doing this, we can find the functional form of  $a_0(i_t)$  and  $a_L(i_t)$ , and the policy function  $N(i_t)$ , which is only a function of  $i_t$ . Equations for these quantities can be found in Appendix A3.

Finally, for the unconstrained bank, it is always the case that

$$Div_t = E_t - \gamma V_t^L(L_t + N_t) + (V_t^L - \mathcal{P}_t)N_t \quad (34)$$

so that it is  $E_t' = \gamma V_t^L(L_t + N_t)$ . There is no point in keeping equity in the bank in excess of the equity requirement. Equity can be raised without frictions next period, and this is more attractive than keeping profits in the firm due to taxes.

## 5 Calibration

I take one period to be a quarter and normalize consumption to be 10 each quarter in steady state, i.e. when the nominal interest rate is equal to the natural rate, taken to be equal to 4% in annual terms. The parameter values are summarized in Table 2. The (discrete) shock distributions are summarized in Table 3. I discretize the interest rate process  $i_t$ : I use the 5 values [2%, 3%, 4%, 5%, 6%]. The transition matrix for the nominal interest rate is consistent with the solution of the system (16), (18), (20), namely it is  $\mathbb{E}_t[i_{t+1} - r_n] = \rho_v(i_t - r_n)$ , and corresponds to a volatility of the monetary shock  $\epsilon^v$  of about 0.57%.

The mean of  $\omega$  (annualized) is 1.3%, close to the historic US average for commercial, industrial and consumer loans. The value of the repayment rate  $\bar{\delta}$ , 25% annualized, implies a loan maturity of 4 years.

The aggregate productivity in the BD sector  $\zeta$ , appearing in the loan demand (23), determines the size of the bank. I calibrate it so that deposits, determined by household demand for liquidity, represent around 70% of the bank's liabilities, in line with averages for US banks (for deposits net of reserve requirements). Without deposits the fixed cost  $c_F$  is calibrated so that the average "market-to-book" ratio of the bank's equity

Table 2: Bank & BD sector parameters	
Parameter	Description
$\gamma = 0.08$	equity requirement
$\tau = 0.35$	corporate tax
$\epsilon_B = 6$	elast.of substit. bank loans
$\nu = 0.33$	capital share BD sector
$\bar{\delta} = 0.25$	loan repayment rate
$\omega = 0.013$	loan default rate
$\bar{\rho} = 0.08$	stand. loans coupon rate (annual)
$\zeta = 0.22$	aggregate productivity BD sector
$c_F^{high-cost} = -0.093$	fixed cost with deposits (high-cost case)
$c_F^{zero-cost} = -0.046$	fixed cost without deposits (& in zero-cost case)

$q \equiv V/E$ , where  $V$  is the bank’s value function, is 1.15, close to an average value for banks in the US.<sup>9</sup> In the model with deposits, I consider two alternative, extreme cases. In one case (“zero-cost”) I assume that managing deposits entails no extra cost. Hence in this case  $c_F$  is the same as in the model without deposits. In another case (“high-cost”) I assume that the cost of managing deposits is such that profits on deposits, net of this cost, are zero on average. In the latter case the advantage for banks of issuing deposits lies entirely in the countercyclical nature of the associated profits.

## 6 Results

### 6.1 The Unconstrained Bank

The numerical value of the value function, the policy function  $N^U$  (where the superscript emphasizes that it is the policy function for the unconstrained bank), and the corresponding loan price  $\mathcal{P}^U = \mathcal{P}(N^U)$  are in Table 4, for the case in which the bank optimally chooses deposits, and in the case in which deposits are 0. Analytic formulas for  $a_0$  and  $a_L$ , as well as for the maximization problem that determines  $N^U$ , can be found in Appendix A3. Notice that deposits affect the term  $a_0$  of the value function,

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<sup>9</sup>For example <https://www.investopedia.com/ask/answers/040815/what-average-pricetobook-ratio-bank.asp> reports that, as January 2015, the average market-to-book ratio for US banks was 1.1.

Table 3: Shocks		
Shock	Values (Annualized)	Probabilities
$\omega$	$[-0.0052, 0.0016, 0.0168, 0.04]$	$[0.25, 0.25, 0.25, 0.25]$
Variable	Values	Transition Matrix
$i$	$[0.02, 0.03, 0.04, 0.05, 0.06]$	$\begin{bmatrix} 0.3 & 0.6 & 0.1 & 0 & 0 \\ 0.05 & 0.5 & 0.45 & 0 & 0 \\ 0 & 0.16 & 0.68 & 0.16 & 0 \\ 0 & 0 & 0.45 & 0.5 & 0.05 \\ 0 & 0 & 0.1 & 0.6 & 0.3 \end{bmatrix}$

but not the policy function  $N^U$ . The intuition is that the ability to raise equity on the market allows the bank to undertake all profitable lending opportunities, regardless of the profits it makes in its deposit activity.

Table 4: Unconstrained Bank					
With Deposits	$i = 2\%$	$i = 3\%$	$i = 4\%$	$i = 5\%$	$i = 6\%$
$N^U(i)$	0.879	0.874	0.869	0.863	0.858
$\mathcal{P}^U(i)$	1.025	1.021	1.017	1.013	1.009
$a_0(i)$	-0.0423	-0.0158	0.0093	0.0331	0.0557
$a_L(i)$	0.0342	0.0340	0.0339	0.0338	0.0336
Without Deposits	$i = 2\%$	$i = 3\%$	$i = 4\%$	$i = 5\%$	$i = 6\%$
$a_0(i)$	0.0102	0.0096	0.0090	0.0084	0.0078

## 6.2 The Constrained Bank: Moments

The bank's problem in the constrained case can be solved with value function iteration methods. As also found by vdH, the value function is highly non-linear, especially



when equity is close to the equity requirement value  $E = \gamma V^L L$ , which implies that linearization techniques are unsuitable for this problem. I discretize the state variables  $L$  and  $E$ , in addition to  $i$ , as discussed in the calibration section, making the grid denser in the region of high non-linearity.

The main moments from the simulation of the solved model are shown in Table 5. The left side of the table shows the moments for the problem without deposits (in which it is simply  $D_t = 0$  for every  $t$ ). The left side shows the moments for the full problem with deposits. Deposits represent on average 71% of total bank liabilities. With deposits, the average number of new loans  $N$  and the stock of all outstanding loans  $L$  increase, and their standard deviations decrease. Equity as a fraction of assets, both at the beginning and at the end of the period, decrease, but their standard deviations decrease. All autocorrelations decrease, reflecting a higher tendency of the main variables to mean-revert. However, the effect of deposits seems very modest if we just look at these moments.

The effect of deposits is however quite sizeable in periods of high interest rates and when equity is low. Notice that, with the interest rate transition matrix shown in Table 2, which is consistent with a persistence of the monetary policy shock  $\rho_v = 0.6$ , the two extreme values of interest rate (2% and 6%) occur with a probability of only 1.5% each, so even if deposits have a big effect on lending for  $i = 6\%$ , this has low impact on averages.

Table 5: Simulated Moments									
Variable	Without Deposits			With Dep. high-cost case			With Dep., zero-cost case		
	mean	std	autocorr	mean	std	autocorr	mean	std	autocorr
E (frac of L)	0.095	0.004	0.60	0.094	0.004	0.55	0.092	0.004	0.117
L	11.80	0.70	0.98	11.90	0.52	0.96	12.05	0.24	0.93
N	0.83	0.083	0.62	0.842	0.070	0.51	0.850	0.044	0.118
E' (frac of L)	0.087	0.0027	0.64	0.087	0.0026	0.57	0.082	0.001	0.25
D	0	0	-	7.88	0.11	0.56	7.88	0.11	0.56
D/(Total Liab)	0	0	-	0.72	0.036	0.94	0.72	0.02	0.78

Table 6: New Loans by Interest Rate					
i	Prob.	$N^U$	$N$ without Dep.	$N$ with Dep., high-cost	$N$ with Dep., low-cost
2%	1.5%	0.879	0.876	0.870	0.873
3%	20%	0.874	0.866	0.863	0.863
4%	57%	0.868	0.841	0.845	0.851
5%	20%	0.863	0.781	0.801	0.826
6%	1.5%	0.858	0.650	0.700	0.742

**Figure 2: New Loans**

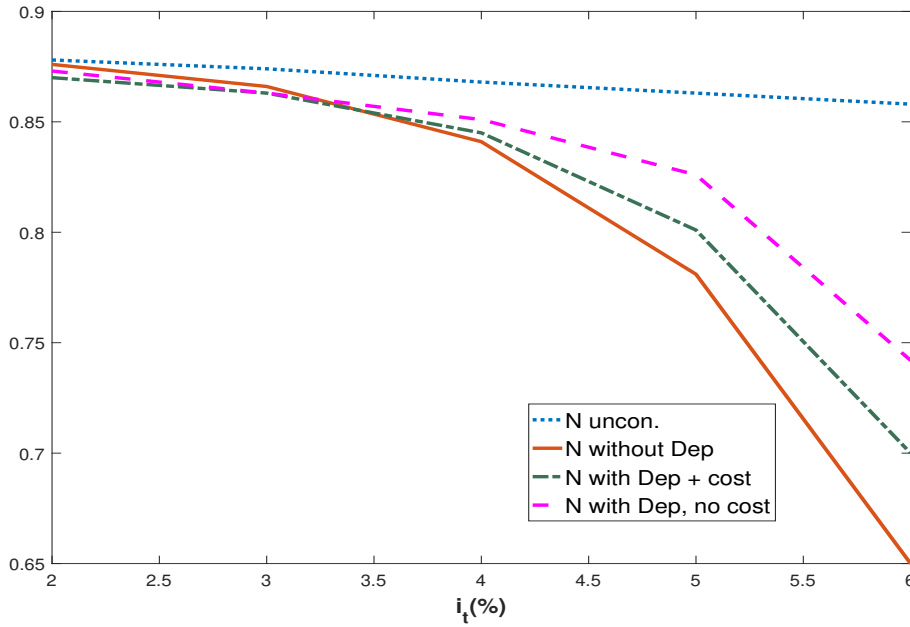


Table 6 and Figure 2 show average new lending for each interest rate value, comparing the unconstrained lending  $N^U$  with the constrained lending with and without deposits, and, in the case with deposits, distinguishing between the case with zero-cost of managing deposits ("zero-cost" case) and the case with cost equal to the average profit on deposits ("high-cost" case). We see that lending above 4% and especially at 6% is dramatically affected by the friction on equity. At 5%, if deposits are 0 new lending is about 9.5% lower than in the unconstrained case, and with deposits the reduction in new loans is only around 4.3% (in the "zero-cost" case) and 7.2% (in the "high-cost"

case). At 6%, if deposits are 0 new lending is about 24.5% lower than in the unconstrained case, and with deposits the reduction in new loans is 13.5% (in the "zero-cost" case) and 18.4% (in the "high-cost" case). In sum, if the bank holds deposits, the reduction in lending at high interest rates is mitigated by a factor roughly between 25% and 50% depending on the cost of managing deposits.

## Discussion

The results obtained in this section imply that, for the average bank that cannot raise equity on the market and that now issues deposits, a narrow banking reform would decrease the issuance of new loans by a percentage between 2.5% and 5.5% after a contractionary monetary shock that bring the policy rate 1% above the natural rate (i.e. to 5% in this model), and between 7.5% and 13% after a shock that bring the policy rate 2% above the policy rate.

The limitation of this model is that, by construction, a contraction in credit supply, which mechanically affects capital and production in the BD sector, has no effect on total output. To have an idea about possible effects of credit supply on output, we have to look beyond this model.

What does the existing literature tell us about the effect of credit supply on total output? The evidence very much depends on the country. Driscoll (2004) finds that for the US the effect of credit supply shocks on output is insignificant. Capiello, Kadareja, Kok and Protopapa (2010), using the same methodology as Driscoll (2004), find that the effect is very significant for countries in the euro area. Specifically, they estimate that a contraction of 1% in the supply of new loans results in a contraction of output by 8 bps.

It is possible that not all banks face the same costs of raising equity. While these costs are probably higher for small and medium banks, the economic intuition of the "leverage ratchet effect" by Admati, Demarzo, Hellwig and Pfleiderer (2018) suggests that shareholders' interests might induce bigger banks to resist equity issuances, even when those would increase the firms' value.

In the limit case in which all banks cannot or do not want to issue equity, and based on the estimates by Capiello, Kadareja, Kok and Protopapa (2010), my results on credit supply suggest that a narrow banking reform in the euro area could reduce output by 20 to 44 bps when the policy rate is 1% above the natural level and by 60

and 104 bps when the policy rate is 2% above the natural level.

### 6.3 The Constrained Bank Problem: Impulse Response Functions at Low Equity

To see the effect of deposits for low equity, I look at impulse-response functions after a contractionary monetary policy shock, using a value of beginning-of-period equity before the shock equal to 8.5% of assets. This low value of beginning-of-period equity occurs about 4% of the time. If the contractionary shock occurs at  $t = \bar{t}$ , the impulse-response function for new lending is defined as

$$IRF_{\bar{t}}^N = E_{\bar{t}}[N_s | i_{\bar{t}} = i_0 + 0.01] - E_{\bar{t}}[N_s | i_{\bar{t}} = i_0], \quad \text{for } s \geq \bar{t} \quad (35)$$

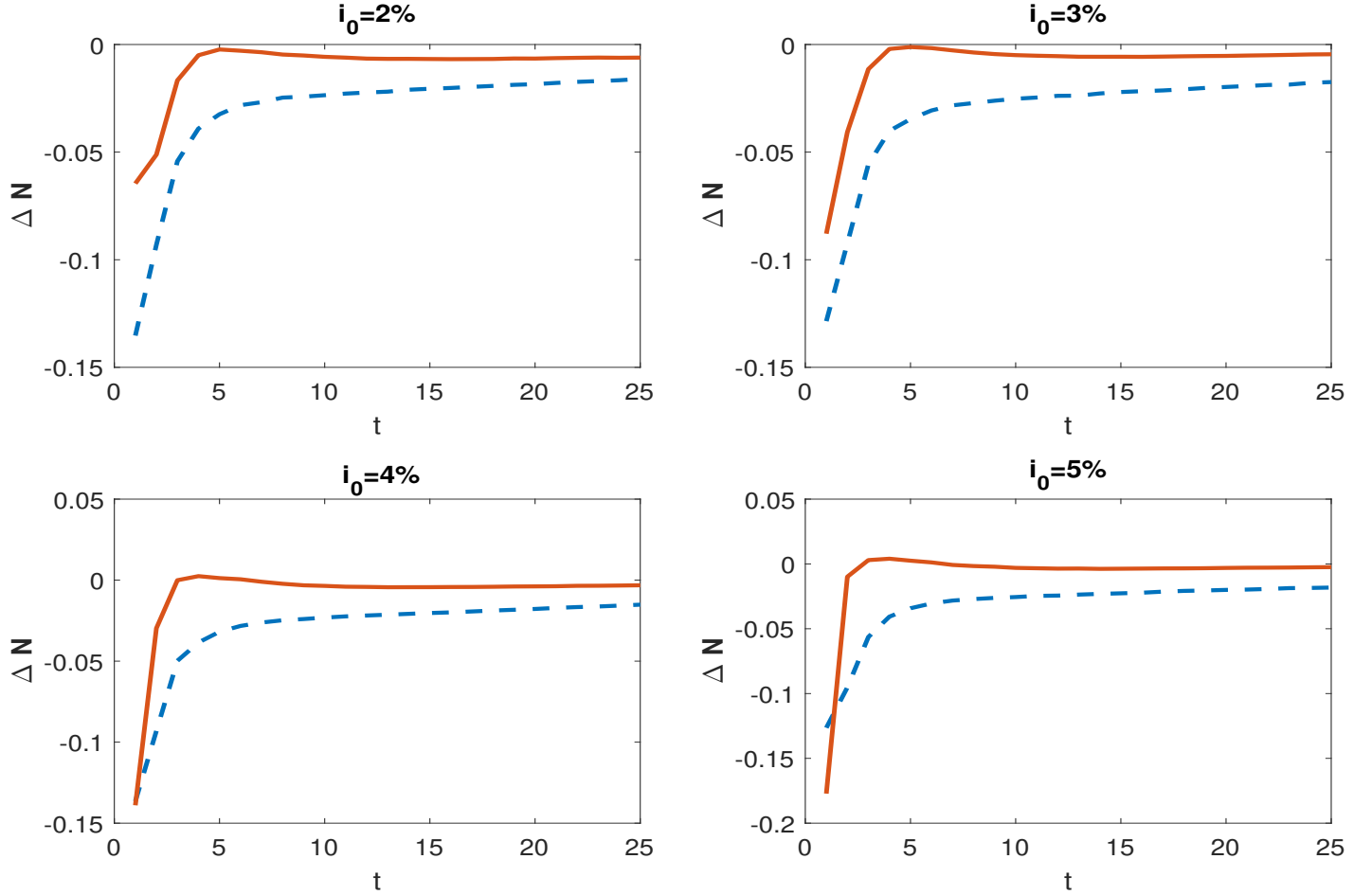
Notice that the non-linearity of the model implies that the impulse response functions cannot be calculated by setting all shocks after the initial one to 0, rather they must be obtained by averaging over all possible future paths.

Figure 3 plots the impulse-response functions (35) for  $i_0 = [2\%, 3\%, 4\%, 5\%]$ , in the model without deposits (solid line), and with deposits in the “high-cost” case (dashed line). For all values of the initial interest rate we notice how the shock has much more persistent effects without deposits. Two effects contribute to this. First, in the case with deposits, the increased profits on deposits after a contractionary shock immediately contribute to rebuilding equity. Second, in the absence of deposits the bank has only one way to rebuild equity: deleveraging, i.e. cutting new lending. However, this means foregoing more profitable lending opportunities, with the effect of slowing down the rebuilding of equity in later periods.

## 7 Conclusion

The results in this paper establish that deposits play an important role in the transmission of monetary policy. As theoretically established by vdH, and empirically confirmed e.g. by Altunbas, Gambacorta and Marques-Ibanez (2011), capital requirements and banks’ balance sheet conditions are important determinants of the supply of loans, especially after contractionary monetary policy shocks.

Figure 3: IRFs



My contribution is to show that profits on deposits mitigate the contraction of credit due to balance sheet effects, and my quantitative exercise finds that this mitigation effect is very significant and increasing in the distance between the policy rate and the natural rate.

This result contradicts the commonly held view, inspired by Romer and Romer (1990), that deposits and any shocks to them do not affect the supply of credit, if the bank can easily switch to alternative forms of financing.

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## Appendix A

### A1: Household FOCs and asset allocation choice

FOC with respect to  $c_t$

$$c_t^{-\sigma} = \lambda_t P_t (1 + \chi(x_t) + x_t \chi'(x_t)) \quad (36)$$

which becomes, specialized to the transaction cost (10)

$$c_t^{-\sigma} = \lambda_t P_t (1 + 2ax_t - 2\sqrt{ab}) \quad (37)$$

FOC with respect to  $h_t$

$$F'(h_t) = \lambda_t W_t \quad (38)$$

FOC with respect to  $A_t$

$$\lambda_t = \beta \lambda_{t+1} (1 + i_t) \quad (39)$$

FOC with respect to  $M_t$

$$\lambda_t \left( 1 - x_t^2 \chi'(x_t) \frac{\partial l_t}{\partial M_t} \right) = \beta \lambda_{t+1} \quad (40)$$

which becomes, specialized to the transaction cost (10)

$$\lambda_t \left( 1 - (ax_t^2 - b) \frac{\partial l_t}{\partial M_t} \right) = \beta \lambda_{t+1} \quad (41)$$

Using the definition of liquidity (5), (40) can be written as

$$\lambda_t \left( 1 - x_t^2 \chi'(x_t) \delta \left( \frac{M_t}{l_t} \right)^{-\frac{1}{\epsilon}} \right) = \beta \lambda_{t+1} \quad (42)$$



FOC with respect to  $D_t$

$$\lambda_t \left( 1 - x_t^2 \chi'(x_t) \frac{\partial l_t}{\partial D_t} \right) = \beta \lambda_{t+1} (1 + i_t^d) \quad (43)$$

which becomes, specialized to the transaction cost (10)

$$\lambda_t \left( 1 - (ax_t^2 - b) \frac{\partial l_t}{\partial D_t} \right) = \beta \lambda_{t+1} (1 + i_t^d) \quad (44)$$

Using (5), (43) can be written as

$$\lambda_t \left( 1 - x_t^2 \chi'(x_t) \left( \frac{D_t}{l_t} \right)^{-\frac{1}{\epsilon}} \right) = \beta \lambda_{t+1} (1 + i_t^d) \quad (45)$$

Combining (39), (42) and (45) I get

$$\frac{1}{\delta} \left( \frac{D_t}{M_t} \right)^{-\frac{1}{\epsilon}} = \frac{i_t - i_t^d}{i_t} \equiv s_t \quad (46)$$

so

$$M_t = (s_t \delta)^\epsilon D_t \quad (47)$$

Hence

$$l_t = D_t (1 + \delta (\delta s_t)^{\epsilon-1})^{\frac{\epsilon}{\epsilon-1}} = D_t f_t \quad (48)$$

with  $f_t \equiv (1 + \delta (\delta s_t)^{\epsilon-1})^{\frac{\epsilon}{\epsilon-1}}$ . Combining (39), (45) and (48) I get

$$f_t^{\frac{1}{\epsilon}} x_t^2 \chi'(x_t) = \frac{i_t - i_t^d}{1 + i_t} \quad (49)$$

which implies, with transaction cost (10),

$$D_t = \frac{P_t c_t}{f_t} \sqrt{\frac{a(1 + i_t)}{f_t^{-\frac{1}{\epsilon}} (i_t - i_t^d) + b(1 + i_t)}} \quad (50)$$

Finally, the three Euler equations, with respect to  $A$ ,  $D$  and  $M$ , respectively, are

$$\frac{c_t^{-\sigma}}{P_t(1 + 2ax_t - 2\sqrt{ab})} = \beta(1 + i_t) E_t \left[ \frac{c_{t+1}^{-\sigma}}{P_{t+1}(1 + 2ax_{t+1} - 2\sqrt{ab})} \right] \quad (51)$$

$$\frac{c_t^{-\sigma} (1 - f_t^{\frac{1}{\epsilon}} (ax_t^2 - b))}{P_t(1 + 2ax_t - 2\sqrt{ab})} = \beta(1 + i_t^d) E_t \left[ \frac{c_{t+1}^{-\sigma}}{P_{t+1}(1 + 2ax_{t+1} - 2\sqrt{ab})} \right] \quad (52)$$

and

$$\frac{c_t^{-\sigma} (1 - \delta ((\delta s_t)^{1-\epsilon} + \delta)^{\frac{1}{\epsilon-1}} (ax_t^2 - b))}{P_t(1 + 2ax_t - 2\sqrt{ab})} = \beta E_t \left[ \frac{c_{t+1}^{-\sigma}}{P_{t+1}(1 + 2ax_{t+1} - 2\sqrt{ab})} \right] \quad (53)$$

## A2: The choice of the deposit rate

The monopolist bank maximizes (13) subject to the deposit demand (12). The first order condition, written in terms of the relative spread  $s_t \equiv \frac{i_t - i_t^d}{i_t}$  is

$$\left(1 - \frac{\epsilon \delta^\epsilon s_t^{\epsilon-1}}{1 + \delta^\epsilon s_t^{\epsilon-1}}\right) + \frac{i_t}{2((1 + \delta^\epsilon s_t^{\epsilon-1})^{-\frac{1}{\epsilon-1}} s_t i_t + b(1 + i_t))} \left(\frac{\delta^\epsilon s_t^\epsilon}{(1 + \delta^\epsilon s_t^{\epsilon-1})^{\frac{\epsilon}{\epsilon-1}}} - \frac{s_t}{(1 + \delta^\epsilon s_t^{\epsilon-1})^{\frac{1}{\epsilon-1}}}\right) = 0 \quad (54)$$

This condition implicitly defines  $s_t$  (or  $i_t^d$ ) as a function of  $i_t$ . For small  $i_t$ , meaning  $i_t \ll b$  we can neglect the second term on the LHS of (54), which can then be written as

$$\left(1 - \frac{\epsilon \delta^\epsilon s_t^{\epsilon-1}}{1 + \delta^\epsilon s_t^{\epsilon-1}}\right) = 0 \quad (55)$$

Hence, for  $i_t \ll b$  it is

$$s_t = \frac{i_t - i_t^d}{i_t} = \left(\frac{1}{(\epsilon - 1)\delta^\epsilon}\right)^{\frac{1}{\epsilon-1}} \quad (56)$$

For high  $i_t$ , meaning  $i_t \gg b$  then (54) can be approximated as

$$\left(1 - \frac{\epsilon \delta^\epsilon s_t^{\epsilon-1}}{1 + \delta^\epsilon s_t^{\epsilon-1}}\right) + \frac{1}{2} \left(\frac{\delta^\epsilon s_t^{\epsilon-1}}{(1 + \delta^\epsilon s_t^{\epsilon-1})} - 1\right) = 0 \quad (57)$$

which implies

$$s_t = \frac{i_t - i_t^d}{i_t} = \left(\frac{1}{2(\epsilon - 1)\delta^\epsilon}\right)^{\frac{1}{\epsilon-1}} \quad (58)$$

## A3: The Unconstrained Value Function

Inserting (33) in (29), and using the law of motion of loans (22), the law of motion of equity (30) and the choice of dividends of the unconstrained bank (34), I find that the vector  $a_L$  (each component of which correspond to a value of  $i$ ) satisfies

$$\begin{aligned} a_L &= (1 - DF \times M)^{-1} [-\gamma V_L + (1 - \bar{\delta} - \bar{\omega}) DF \times M \times V_L + (\bar{\delta} + \tau \bar{\omega}) DF_v \\ &+ (1 - \tau)(1 - \bar{\omega}) \bar{\rho} DF_v - (1 - \gamma) DF \times (V_L + (1 - \tau)(V_L * i))] \end{aligned} \quad (59)$$

where  $DF$  is the square diagonal matrix  $n \times n$  (if the vector  $i$  has  $n$  elements) with the  $n$  discount factors  $\frac{1}{1+i}$  on the diagonal,  $DF_v$  is an  $n$ -component vector equal to the diagonal of  $DF$ ,  $M$  is the transition matrix for the risk-free rate  $i$  (see Table 3),  $\bar{\omega}$  is the average value of the shock  $\omega$ ,  $V_L$  is the  $n$ -component vector of loan values (see (??)) for each value of  $i$ . I denote by  $a * b$  the element-by element product of two vectors  $a$  and  $b$ , so that for example  $V_L * i$  is the element-by-element product of  $V_L$  and  $i$ .

$N^U$  is the  $n$ -component vector that maximizes

$$\begin{aligned} N^U * [(1 - \gamma)V_L + (1 - \bar{\delta} - \bar{\omega})DF \times M \times a_L + (1 - \bar{\delta} - \bar{\omega})DF \times M \times V_L \\ - (1 - \gamma)DF \times ((1 - (1 - \tau)i) * V_L) + ((1 - \tau)(1 - \bar{\omega})\bar{\rho} + \bar{\delta} + \tau\bar{\omega})M \times (1 + \pi)] \\ - \mathcal{P}^U N^U \end{aligned} \quad (60)$$

where  $\pi$  is the  $n$ -component inflation vector and  $\mathcal{P}^U$  is the  $n$ -component price vector. Finally

$$\begin{aligned} a_0 &= (1 - DF \times M)^{-1} [\tau DF \times (V_L * i * N^U) - \mathcal{P}^U * N^U \\ &+ (1 - \bar{\delta} - \bar{\omega})(DF \times M \times a_L) * N^U + DF \times ((1 - \tau)(i - i^d) * D \\ &+ \bar{\delta} * N^U + (1 - \bar{\omega})\bar{\rho}N^U + (1 - \tau)c^F + \tau\bar{\omega}N^U)] \end{aligned} \quad (61)$$

Notice that  $a_0$  is the only quantity which depends on deposits  $D$ .

## Appendix B: The Macroeconomic Environment

There are two firm sectors in the economy, corresponding to two technologies for the production of the consumption good. One sector, the *NK sector*, is typical of a standard New-Keynesian model and generates the NKPC. Notice that NK firms use only labor as factor of production and do not need to borrow (either from households or from banks). The other sector, the *BD sector* is comprised of firms working on long-term projects, that need to borrow from banks. BD firms therefore generate loan demand.

### B1: The NK sector

As in the basic NK model (e.g. Gali(2008)), there are intermediate good producers and final good producers. Intermediate good producers are a continuum of firms indexed by  $i \in [0, 1]$ . Each firm  $i$  produces a different variety of intermediate good, using only labor as input, according to the production function

$$Y_t(i) = aN_t(i)^{1-\alpha} \quad (62)$$

where  $a$  represents the (constant) level of technology and  $N_t$  is labor at time  $t$ . I take  $\alpha = 0$ . This choice, together with the choice of linear labor disutility, results in

a New-Keynesian Phillips curve that is independent of the banking sector, as shown below.

Each firm produces a differentiated good and is a price setter for that good. However, following Calvo (1983), each firm is able to reset its price only with probability  $1 - \theta$  in any given period. Thus each period a fraction  $\theta$  of firms keeps their price unchanged.

Final good producers are perfectly competitive firms taking the different varieties of intermediate product as input. The production function for the final good is

$$Y_t^{NK} = \left( \int di \, y_{iy}^{\frac{\epsilon_G - 1}{\epsilon_G}} \right)^{\frac{\epsilon_G}{\epsilon_G - 1}} \quad (63)$$

where the superscript indicates that this is the final production of the  $NK$  sector and  $\epsilon_G$  is the elasticity of substitution between different varieties of intermediate goods. The final good producer's problem is

$$\max_{Y_t, y_{j,t}, i \in [0,1]} P_t Y_t - \int_0^1 di \, p_{it} y_{it} \quad (64)$$

where  $y_{jt}$  is the demanded quantity of the intermediate good  $j$ . From the first-order conditions, the demand for the intermediate good of variety  $i$  is

$$y_{it} = \left( \frac{p_{jt}}{P_t} \right)^{-\epsilon_G} Y_t \quad (65)$$

Moreover, from the zero-profit condition, the price for the final good is given by  $P_t = \left( \int_0^1 p_{it}^{1-\epsilon_G} di \right)^{\frac{1}{1-\epsilon_G}}$

As in Gali (2008), the fraction  $1 - \theta$  of intermediate-good producers who can reset their price at time  $t$  need to solve the intertemporal problem of choosing the price that maximizes the present value of profits from the current period to the next period they will be able to reset their price, discounted with the household stochastic discount factor. Gali (2008) shows that this problem leads to the New-Keynesian Phillips curve, which in its log-linear form reads

$$\pi_t = \beta E_t[\pi_{t+1}] + \Lambda \hat{m}c_t \quad (66)$$

where  $\hat{m}c$  is the log-deviation of the marginal cost from steady state, and

$$\Lambda = \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \Theta \quad (67)$$

$$\Theta = \frac{1 - \alpha}{1 - \alpha + \alpha \epsilon_G} \quad (68)$$

Again, Gali (2008) shows that marginal cost can be written as

$$mc_t = \sigma c_t + (\phi + \alpha)n_t - \log(a) - \log(1 - \alpha) \quad (69)$$

Notice that, if both  $\phi$  and  $\alpha$  are equal to 0, i.e. if both the production function and the disutility of labor are linear in labor, marginal cost (69) is independent of labor. Since I assume constant productivity  $a$ , I have

$$\hat{m}c = \sigma \hat{c} \quad (70)$$

and the New-Keynesian Phillips curve is

$$\pi_t = \beta E_t[\pi_{t+1}] + \Lambda \hat{c}_t \quad (71)$$

where  $\hat{c}$  is the log-deviation of consumption from steady state.

## B2: Unit Loans

A unit loan  $K_t$ , granted by a bank to a firm at time  $t$ , starts with the transfer of one unit of good from the bank to the firm at time  $t$ . A loan rate  $\rho^t$  is also established at time  $t$ . A unit loan demands a payment  $(\bar{\delta} + \rho^{(t)})(1 - \bar{\delta})^{n-1}$  at each time  $t + n$ ,  $n = 1, \dots, \infty$ . In reality, a fraction  $\omega_{t+n}$  defaults at each time  $t + n$ , resulting in actual payments

$$\begin{aligned} & (\bar{\delta} + \rho^{(t)})(1 - \omega_{t+1}) & \text{at } t + 1 \\ & (\bar{\delta} + \rho^{(t)})(1 - \omega_{t+2})(1 - \bar{\delta} - \omega_{t+1}) & \text{at } t + 2 \\ & (\bar{\delta} + \rho^{(t)})(1 - \omega_{t+3})(1 - \bar{\delta} - \omega_{t+2})(1 - \bar{\delta} - \omega_{t+1}) & \text{at } t + 3 \\ & \dots & \\ & (\bar{\delta} + \rho^{(t)})(1 - \omega_{t+n})(1 - \bar{\delta} - \omega_{t+n-1}) \dots (1 - \bar{\delta} - \omega_{t+2})(1 - \bar{\delta} - \omega_{t+1}) & \text{at } t + n \end{aligned} \quad (72)$$

where the shocks  $\omega$  are i.i.d and independent of the monetary shock. Notice that the cashflows (72) are proportional to the cashflow (21) of standard loans defined in Section 4, with proportionality factor  $\frac{\bar{\delta} + \rho^{(t)}}{\bar{\delta} + \rho}$ . Hence one unit loan issued at time  $t$  at interest

$\rho_t$  is equivalent to  $\frac{\bar{\delta} + \rho^{(t)}}{\bar{\delta} + \bar{\rho}}$  “standard” loans, as defined in section 4. Since the price of a unit loan is 1 by construction, the price of a standard loan must then be

$$\mathcal{P}_t = \frac{\bar{\delta} + \bar{\rho}}{\bar{\delta} + \rho^{(t)}} \quad (73)$$

The advantage of standard loans as defined in Section 4 is that they have more convenient aggregation properties: one loan issued at  $t$  becomes equivalent, at  $t + 1$ , to  $1 - \delta_K$  loans issued at  $t + 1$ . In contrast, unit loans  $K$  issued at time  $t$  are equivalent, at  $t + 1$ , to a number of loans that depends on  $\rho^{(t+1)}$  and  $\rho^{(t)}$ .

### B3: The BD sector

Each period a new time- $t$  representative start-up firm installs capital borrowed from different banks.

Production for the time- $t$  start-up occurs starting at  $t + 1$ , with the only factor of production being the capital installed at  $t$ :

$$\begin{aligned} Y_{t+1}^{(t)} &= \zeta \tilde{K}_t^\nu \\ Y_{t+2}^{(t)} &= \zeta (1 - \delta_K)^\nu \tilde{K}_t^\nu \\ &\dots \dots \\ Y_{t+s}^{(t)} &= \zeta (1 - \delta_K)^{(s-1)\nu} \tilde{K}_t^\nu \end{aligned}$$

with  $\tilde{K}_t = \left( \int_0^1 di (K_t^{(i)})^{\frac{\epsilon_B - 1}{\epsilon_B}} \right)^{\frac{\epsilon_B}{\epsilon_B - 1}}$ , where  $K_t^{(i)}$  are the units of capital loaned by bank  $i$  at  $t$  and  $\epsilon_B$  is the elasticity of substitution of loans from different banks.  $\delta_K$  is the depreciation rate of capital.

For each unit loan obtained from bank  $i$  at  $t$ , if the loan rate is  $\rho_i$  the firm owes  $(1 - \bar{\delta})^{s-1}(\rho_i + \bar{\delta})$  in each subsequent period  $t + s$ .

Therefore the firm’s profits at time  $t + 1$  are

$$\Pi_{t+1} = \zeta \tilde{K}_t^\nu - \int di (\rho_i + \bar{\delta}) K_t^{(i)} \quad (74)$$

and in each subsequent period  $s$

$$\Pi_{t+s} = \zeta (1 - \delta_K)^{\nu(s-1)} \tilde{K}_t^\nu - (1 - \bar{\delta})^{s-1} \int di (\rho_i + \bar{\delta}) K_t^{(i)} \quad (75)$$

Assuming that the firm negotiated the repayment rate  $\bar{\delta}$  to the bank so that  $(1 - \delta_K)^\nu = (1 - \bar{\delta})$ , then each period's profits are simply scaled down by a factor  $(1 - \bar{\delta})$  relative to those of the previous period. Hence the loan decision that maximizes the first period's profits also maximizes the profits of each subsequent period.

The profit-maximizing capital borrowed at  $t$  from bank  $i$  is

$$K_t^{(i)} = (\zeta\nu)^{\epsilon_B} (\rho_i + \bar{\delta})^{-\epsilon_B} \tilde{K}_t^{1-\epsilon_B(1-\nu)} \quad (76)$$

After some straightforward algebra I obtain

$$\tilde{K}_t = (\zeta\nu)^{\frac{1}{1-\nu}} (\rho_M + \bar{\delta})^{-\frac{1}{1-\nu}} \quad (77)$$

with

$$(\rho_M + \bar{\delta}) = \left( \int di (\rho_i + \bar{\delta})^{1-\epsilon_B} \right)^{\frac{1}{1-\epsilon_B}} \quad (78)$$

Substituting (77) in (76) I get

$$K_t^{(i)} = (\zeta\nu)^{\frac{1}{1-\nu}} (\rho_i + \bar{\delta})^{-\epsilon_B} (\rho_M + \bar{\delta})^{-\frac{1}{1-\nu} + \epsilon_B} \quad (79)$$

In terms of standard loans, where bank  $i$  makes an initial payment  $\mathcal{P}^i = \frac{\bar{\delta} + \bar{\rho}}{\bar{\delta} + \rho_i}$  to the firm and the loan rate is a constant  $\bar{\rho}$ , (standardized) loan demand is

$$N^i = (\zeta\nu)^{\frac{1}{1-\nu}} (\bar{\rho} + \bar{\delta})^{-\frac{1}{1-\nu}} (\mathcal{P}^i)^{\epsilon_B - 1} (\mathcal{P}^M)^{\frac{1}{1-\nu} - \epsilon_B} \quad (80)$$

where the market price  $\mathcal{P}^M$  is related to  $\rho_M$  by  $\mathcal{P}^M = \frac{\bar{\delta} + \bar{\rho}}{\bar{\delta} + \rho_M}$ .